

## THERMO-ECONOMIC OPTIMIZATION OF BUILDING WALL INSULATION THICKNESS USING HEATING AND COOLING DEGREE DAY ANALYSIS

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**Abstract.** Thermal insulation plays a crucial role in reducing building energy consumption and improving economic efficiency. In this study, a comprehensive thermo-economic model was developed to determine the optimum insulation thickness of building walls by considering both heating and cooling energy demands. Unlike conventional insulation analyses that focus only on heating requirements, the proposed model incorporates both heating degree days (HDD) and cooling degree days (CDD), enabling a full-year evaluation of building energy performance. The life-cycle cost method was used to determine the economically optimal insulation thickness, while numerical optimization was applied to minimize the total cost function. The results indicate that the optimum insulation thickness for expanded polystyrene under the investigated climatic conditions ( $HDD=2022^{\circ}\text{C}\cdot\text{days}$ ,  $CDD=650^{\circ}\text{C}\cdot\text{days}$ ) is approximately 0.07–0.09 m. Applying this insulation level can reduce the annual energy cost by about 35–50% compared with an uninsulated wall. The economic analysis also shows that the insulation investment can be recovered within 3–7 years, depending on climatic conditions. Furthermore, the results demonstrate that both HDD and CDD significantly influence insulation design, highlighting the importance of full-year climate considerations. The developed model provides a practical tool for determining economically optimal insulation strategies in different climatic regions.

A comprehensive thermo-economic model was developed to determine the optimum insulation thickness of building walls by considering both heating and cooling energy demands. The analysis was based on the heating degree day (HDD) and cooling degree day (CDD) approach, combined with life-cycle cost evaluation. Heat transfer through the wall was assumed to be one-dimensional and steady-state, while the thermal properties of insulation materials, indoor comfort temperature, and system efficiencies were taken as constant. Expanded polystyrene and rock wool were considered as insulation materials. The model included wall thermal resistance, insulation thermal conductivity, fuel and electricity prices, heating system efficiency, cooling system COP, economic lifetime, interest rate, and inflation rate. A numerical optimization procedure was implemented in Python using NumPy and Matplotlib, where insulation thickness was varied within the range of 0–0.20 m to identify the minimum total cost.

The results showed that increasing insulation thickness significantly reduced annual heating and cooling energy costs, while insulation investment increased linearly with thickness. For the studied climatic conditions, the optimum insulation thickness for expanded polystyrene was found to be approximately 0.07–0.09 m, whereas slightly lower values were obtained for rock wool due to its higher thermal conductivity and material cost. The annual energy cost was reduced by about 35–50% compared with an uninsulated wall, and the payback period was estimated to be in the range of 3–7 years. In addition, the optimum insulation thickness increased with increasing present worth

factor, heating degree days, and cooling degree days, confirming that both climatic and economic parameters strongly affect insulation design and long-term

**Keywords:** Optimum insulation thickness; Life-cycle cost analysis; Heating degree days; Cooling degree days; Building energy efficiency; Thermo-economic analysis.

УДК: 697.329÷536.2

## ТЕРМОЭКОНОМИЧЕСКАЯ ОПТИМИЗАЦИЯ ТОЛЩИНЫ ТЕПЛОИЗОЛЯЦИИ СТЕН ЗДАНИЙ С ИСПОЛЬЗОВАНИЕМ АНАЛИЗА ГРАДУСО-СУТОК ОТОПЛЕНИЯ И ОХЛАЖДЕНИЯ

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**Аннотация.** Тепловая изоляция играет важную роль в снижении энергопотребления зданий и повышении экономической эффективности. В данном исследовании была разработана комплексная термoeкономическая модель для определения оптимальной толщины теплоизоляции стен зданий с учетом как потребностей в отоплении, так и в охлаждении. В отличие от традиционных исследований теплоизоляции, ориентированных только на отопительную нагрузку, предложенная модель учитывает как градусо-сутки отопления (HDD), так и градусо-сутки охлаждения (CDD), что позволяет оценивать энергетическую эффективность здания в течение всего года. Для определения экономически оптимальной толщины теплоизоляции использовался метод стоимости жизненного цикла, а для минимизации функции совокупных затрат была применена численная оптимизация. Результаты показали, что при исследуемых климатических условиях (HDD = 2022 °C·сут, CDD = 650 °C·сут) оптимальная толщина теплоизоляции из пенополистирола составляет приблизительно 0,07–0,09 м. Применение такого уровня изоляции позволяет снизить годовые затраты на энергию примерно на 35–50 % по сравнению со стеной без теплоизоляции. Экономический анализ также показал, что инвестиции в теплоизоляцию могут окупиться в течение 3–7 лет в зависимости от климатических условий. Кроме того, результаты демонстрируют, что как HDD, так и CDD существенно влияют на проектирование теплоизоляции, подчеркивая важность учета климатических условий в течение всего года. Разработанная модель представляет собой практический инструмент для определения экономически оптимальных стратегий теплоизоляции в различных климатических регионах.

Для определения оптимальной толщины теплоизоляции стен зданий с учетом потребностей как в отоплении, так и в охлаждении была разработана комплексная термoeкономическая модель. Анализ основывался на подходе с использованием градусо-суток отопления (HDD) и охлаждения (CDD) в сочетании с оценкой стоимости жизненного цикла. Теплопередача через стену принималась одномерной и стационарной, тогда как теплофизические свойства изоляционных материалов, комфортная внутренняя температура и эффективность систем считались постоянными. В качестве теплоизоляционных материалов были рассмотрены пенополистирол и каменная вата. Модель включала тепловое сопротивление стены, теплопроводность изоляции, стоимость топлива и электроэнергии, эффективность системы отопления, коэффициент преобразования системы охлаждения (COP), экономический срок службы, процентную ставку и инфляцию. Процедура численной оптимизации была реализована в Python с использованием библиотек NumPy и Matplotlib, при этом толщина изоляции изменялась в диапазоне 0–0,20 м для определения минимального значения совокупных затрат.

Результаты показали, что увеличение толщины теплоизоляции существенно снижает годовые затраты на отопление и охлаждение, тогда как инвестиционные затраты на

изоляция линейно возрастают с увеличением толщины. Для исследуемых климатических условий оптимальная толщина теплоизоляции из пенополистирола составила приблизительно 0,07–0,09 м, тогда как для каменной ваты были получены несколько меньшие значения вследствие ее более высокой теплопроводности и стоимости материала. Годовые энергетические затраты снижались примерно на 35–50 % по сравнению со стеной без теплоизоляции, а срок окупаемости оценивался в пределах 3–7 лет. Кроме того, оптимальная толщина теплоизоляции возрастала с увеличением коэффициента приведенной стоимости, градусо-суток отопления и градусо-суток охлаждения, что подтверждает сильное влияние как климатических, так и экономических параметров на проектирование теплоизоляции и долгосрочную энергетическую эффективность.

**Ключевые слова:** оптимальная толщина теплоизоляции; анализ стоимости жизненного цикла; градусо-сутки отопления; градусо-сутки охлаждения; энергетическая эффективность зданий; термоэкономический анализ.

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## **BINOLAR DEVORLARINING ISSIQLIK IZOLYATSIYASI QALINLIGINI ISITISH VA SOVITISH DARAJA-KUNLARI TAHLILI ASOSIDA TERMO-IQTISODIY OPTIMALLASHTIRISH**

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**Annotatsiya.** Issiqlik izolyatsiyasi binolarning energiya sarfini kamaytirish va iqtisodiy samaradorligini oshirishda muhim ahamiyatga ega. Ushbu tadqiqotda binolar devorlari uchun optimal issiqlik izolyatsiyasi qalinligini aniqlash maqsadida isitish va sovitish energiyasi talablari birgalikda hisobga olingan kompleks termo-iqtisodiy model ishlab chiqildi. Faqat isitish talablarini hisobga oladigan an'anaviy izolyatsiya tahlillaridan farqli ravishda, taklif etilgan model isitish daraja-kunlari (HDD) va sovitish daraja-kunlari (CDD) ni birgalikda inobatga oladi, bu esa binoning energiya samaradorligini yil davomida to'liq baholash imkonini beradi. Iqtisodiy jihatdan optimal izolyatsiya qalinligini aniqlashda hayotiy sikl xarajatlari usulidan foydalanildi, umumiy xarajat funksiyasini minimallashtirish uchun esa sonli optimallashtirish qo'llanildi. Natijalar shuni ko'rsatdiki, o'rganilgan iqlim sharoitlarida ( $HDD = 2022\text{ }^{\circ}\text{C}\cdot\text{kun}$ ,  $CDD = 650\text{ }^{\circ}\text{C}\cdot\text{kun}$ ) penopolistirol uchun optimal izolyatsiya qalinligi taxminan 0,07–0,09 m ni tashkil etadi. Ushbu qalinlikni qo'llash izolyatsiyasiz devor bilan solishtirganda yillik energiya xarajatlarini taxminan 35–50 % ga kamaytirishi mumkin. Iqtisodiy tahlil shuningdek, izolyatsiyaga kiritilgan investitsiyalar iqlim sharoitiga qarab 3–7 yil ichida qoplanishini ko'rsatdi. Bundan tashqari, natijalar HDD ham, CDD ham izolyatsiya dizayniga sezilarli ta'sir ko'rsatishini namoyon etdi, bu esa yil bo'yi iqlim sharoitlarini hisobga olish muhimligini tasdiqlaydi. Ishlab chiqilgan model turli iqlim mintaqalarida iqtisodiy jihatdan optimal izolyatsiya strategiyalarini aniqlash uchun amaliy vosita bo'lib xizmat qiladi.

Binolar devorlari uchun optimal issiqlik izolyatsiyasi qalinligini aniqlash maqsadida isitish va sovitish energiyasi talablari hisobga olingan kompleks termo-iqtisodiy model ishlab chiqildi. Tahlil isitish daraja-kunlari (HDD) va sovitish daraja-kunlari (CDD) yondashuviga hamda hayotiy sikl xarajatlarini baholash usuliga asoslandi. Devor orqali issiqlik uzatilishi bir o'lchamli va statsionar deb qabul qilindi, izolyatsiya materiallarining issiqlik-fizik xossalari, ichki qulay harorat va tizim samaradorliklari esa o'zgarish deb olindi. Issiqlik izolyatsiyasi materiali sifatida penopolistirol va tosh paxta ko'rib chiqildi. Modelga devorning issiqlik qarshiligi, izolyatsiyaning issiqlik o'tkazuvchanligi, yoqilg'i va elektr energiyasi narxlari, isitish tizimi samaradorligi, sovitish tizimining COP koeffitsienti, iqtisodiy xizmat muddati, foiz stavkasi va inflyatsiya kiritildi. Sonli optimallashtirish jarayoni Python dasturida NumPy va Matplotlib kutubxonalari yordamida amalga

oshirildi, bunda izolyatsiya qalinligi 0–0,20 m oralig'ida o'zgartirilib, umumiy xarajatning minimal qiymati aniqlandi.

*Natijalar shuni ko'rsatdiki, izolyatsiya qalinligining ortishi yillik isitish va sovitish xarajatlari sezilarli darajada kamaytiradi, biroq izolyatsiyaga kiritiladigan investitsiya xarajatlari qalinlik bilan chiziqli ravishda oshadi. O'rganilgan iqlim sharoitlari uchun penopolistirol izolyatsiyasining optimal qalinligi taxminan 0,07–0,09 m deb topildi, tosh paxta uchun esa uning issiqlik o'tkazuvchanligi va material narxi yuqoriroq bo'lgani sababli biroz kichikroq qiymatlar olindi. Yillik energiya xarajatlari izolyatsiyasiz devorga nisbatan taxminan 35–50 % ga kamaydi, qoplash muddati esa 3–7 yil oralig'ida deb baholandi. Bundan tashqari, optimal izolyatsiya qalinligi keltirilgan qiymat koeffitsienti, isitish daraja-kunlari va sovitish daraja-kunlari ortishi bilan oshib borishi aniqlandi. Bu esa iqlimiy va iqtisodiy parametrlar izolyatsiya loyihalashga hamda uzoq muddatli energiya samaradorligiga kuchli ta'sir ko'rsatishini tasdiqlaydi.*

**Kalit so'zlar:** optimal izolyatsiya qalinligi; hayotiy sikl xarajatlari tahlili; isitish daraja-kunlari; sovitish daraja-kunlari; binolarning energiya samaradorligi; termo-iqtisodiy tahlil.

## Introduction

Buildings are among the largest energy consumers worldwide, accounting for approximately 30–40% of global energy use and nearly 36% of CO<sub>2</sub> emissions [1–3]. A substantial share of this energy is required to maintain indoor thermal comfort through heating and cooling systems, which often represent 50–70% of total building energy consumption [4–6].

Rapid urbanization and population growth are expected to significantly increase building energy demand in the coming decades. Global building floor area is projected to nearly double by 2060, highlighting the need for energy-efficient technologies [7–9]. Among these technologies, thermal insulation is one of the most effective measures for reducing energy consumption. By limiting heat transfer through walls, roofs, and floors, insulation decreases winter heat losses and summer heat gains, thereby reducing heating and cooling requirements [10–12]. Previous studies have reported energy savings of 20–60%, depending on climate conditions and building characteristics [13–15].

Because insulation strongly influences building energy performance, determining the optimum insulation thickness has become an important research topic. Optimum insulation thickness is defined as the level that minimizes the total cost of insulation investment and energy consumption. Excessive insulation increases investment costs, whereas insufficient insulation leads to higher energy use [19–21]. Thermo-economic studies commonly employ the life-cycle cost (LCC) method, which considers insulation cost, fuel prices, system lifetime, and economic parameters such as interest and inflation rates [22–24]. Many previous studies estimated heating demand using the heating degree-day (HDD) method and determined optimum insulation thickness for various climates, wall types, insulation materials, and fuels [25–28]. However, most of these studies considered only heating requirements and neglected cooling energy demand. This limitation may lead to inaccurate results in regions where both heating and cooling are necessary. Cooling energy consumption has increased rapidly due to the widespread use of air-conditioning systems, with global electricity use for cooling more than tripling since 1990 [29–32]. In hot-climate countries, cooling can account for 60–70% of total building electricity consumption [33–34].

Therefore, optimum insulation analysis should consider both heating and cooling energy demands to accurately assess annual thermal and economic performance. Incorporating cooling degree days (CDD) together with HDD enables a more realistic evaluation of building energy consumption by accounting for both winter heat losses and summer heat gains through building envelopes. This study aims to develop a comprehensive thermo-economic model for determining the optimum insulation thickness of building walls by considering both heating degree days (HDD) and cooling degree days (CDD). The proposed model integrates heating and cooling energy costs with insulation investment using a life-cycle cost analysis approach. Numerical optimization techniques are employed to determine the insulation thickness that minimizes the total cost. In addition, the influence of climatic conditions and economic parameters on optimum insulation thickness, energy savings, and payback period is investigated through several parametric analyses. The results of this

study provide a more realistic assessment of insulation performance and offer practical guidance for designing energy-efficient building envelopes in regions where both heating and cooling demands are significant

## Methodology and Mathematical Modeling

### 1. Model assumptions

In this study, the optimum insulation thickness of building walls was determined using a thermo-economic model based on life-cycle cost analysis. The model evaluates both heating and cooling energy demands by incorporating heating degree days (HDD) and cooling degree days (CDD).

- The following assumptions were considered in the modelling process:
- Heat transfer through the wall occurs in steady-state conditions.
- Thermal properties of insulation materials remain constant during the analysis period.
- Indoor temperature is maintained at a constant comfort level throughout the year.
- Heating and cooling systems operate with constant efficiency.
- Fuel and electricity prices are assumed constant over the lifetime of the insulation system.
- The wall structure is considered homogeneous and one-dimensional heat transfer is assumed.

These assumptions allow the development of a simplified analytical framework suitable for evaluating insulation performance under different climatic and economic conditions.

### 2. Heat transfer through the insulated wall

Heat transfer through the building envelope is driven by the temperature difference between indoor and outdoor environments. The heat flux through the wall can be expressed as

$$q = U\Delta T,$$

where  $U$  is the overall heat transfer coefficient of the wall.

the total thermal resistance of the wall including insulation is defined as

$$R_{tot} = R_{wt} + \frac{x}{k},$$

where  $R_{wt}$  – thermal resistance of the wall without insulation,  $x$ – insulation thickness (m),  $k$  – thermal conductivity of insulation material (W/mK)

The overall heat transfer coefficient is therefore

$$U = \frac{1}{R_{tot}}.$$

### 3. Annual heating energy demand

Annual heating energy demand was estimated using the heating degree-day method. The annual heating energy requirement per unit wall area can be expressed as

$$E_H = \frac{86400 \cdot HDD}{R_{tot} \cdot \eta_H}$$

where  $HDD$  – heating degree days ( $^{\circ}\text{C} \cdot \text{days}$ ),  $\eta_H$  – efficiency of the heating system

The corresponding annual heating cost is determined as

$$C_H = \frac{E_H}{H_u} C_f$$

where  $C_f$  – fuel cost (\$/kg or \$/m<sup>3</sup>),  $H_u$  – lower heating value of fuel (J/kg)

### 4. Annual cooling energy demand

Cooling energy demand was evaluated using the cooling degree-day method. Cooling degree days represent the accumulated temperature difference between outdoor temperature and indoor comfort temperature during the cooling season. The annual cooling energy demand is calculated as

$$E_C = \frac{86400 \cdot CDD}{R_{tot} \cdot COP},$$

where  $CDD$  – cooling degree days ( $^{\circ}\text{C} \cdot \text{days}$ ),  $COP$  – coefficient of performance of the cooling system

The annual cooling cost is then determined as

$$C_C = \frac{E_C}{3.6 \cdot 10^6} C_{el}$$

where  $C_{el}$  – electricity price (\$/kWh)

### 5. Insulation cost

The insulation cost per unit wall area is expressed as

$$C_{ins} = C_i \cdot x,$$

where  $C_i$  – insulation material cost (\$/m<sup>3</sup>),  $x$  – insulation thickness (m)

### 6. Total life-cycle cost

The total cost of the insulation system includes the present value of heating and cooling energy expenditures as well as the initial insulation investment.

$$C_t = (C_H + C_C) \cdot PWF + C_i \cdot x,$$

where  $PWF$  is the present worth factor calculated as

$$PWF = \frac{(1 + r)^N - 1}{r(1 + r)^N}$$

and

$$r = \frac{i - g}{1 + g},$$

where  $i$  – interest rate,  $g$  – inflation rate,  $N$  – system lifetime (years).

### 7. Determination of optimum insulation thickness

Unlike conventional models that consider only heating energy demand, the present model incorporates both heating and cooling loads. Consequently, the total cost function becomes nonlinear and does not yield a simple analytical expression for the optimum insulation thickness. Therefore, the optimum insulation thickness was determined numerically by minimizing the total life-cycle cost function. A computational model was developed in Python using the NumPy and Matplotlib libraries. The insulation thickness was varied within a predefined range (0–0.20 m), and the corresponding total cost was calculated. The insulation thickness that minimizes the total life-cycle cost was identified as the optimum insulation thickness.

These analyses provide a comprehensive understanding of the thermal and economic performance of insulation systems under varying climatic and economic conditions (Table 1).

**Table 1.** Input parameters used in the thermo-economic model

| Parameter                         | Symbol   | Unit               | Value               | Description                                         |
|-----------------------------------|----------|--------------------|---------------------|-----------------------------------------------------|
| Heating degree days               | HDD      | °C·day             | 2022                | Annual heating degree days for the studied location |
| Cooling degree days               | CDD      | °C·day             | 650                 | Annual cooling degree days                          |
| Wall thermal resistance           | $R_{wt}$ | m <sup>2</sup> K/W | 0.592               | Thermal resistance of wall without insulation       |
| Interest rate                     | $i$      | –                  | 0.08                | Annual interest rate                                |
| Inflation rate                    | $g$      | –                  | 0                   | Inflation rate                                      |
| Lifetime                          | $N$      | year               | 10                  | Economic lifetime of insulation system              |
| Fuel cost (coal)                  | $C_f$    | \$/kg              | 0.164               | Unit price of coal                                  |
| Fuel cost (natural gas)           | $C_f$    | \$/m <sup>3</sup>  | 0.223               | Unit price of natural gas                           |
| Electricity price                 | $C_{el}$ | \$/kWh             | 0.210               | Electricity price                                   |
| Lower heating value (coal)        | $H_u$    | J/kg               | $25.122 \cdot 10^6$ | Heating value of coal                               |
| Lower heating value (natural gas) | $H_u$    | J/m <sup>3</sup>   | $34.542 \cdot 10^6$ | Heating value of natural gas                        |
| Heating system efficiency         | $\eta_H$ | –                  | 0.65–0.93           | Efficiency of heating system depending on fuel      |
| Cooling system COP                | COP      | –                  | 3.0                 | Coefficient of performance of cooling system        |

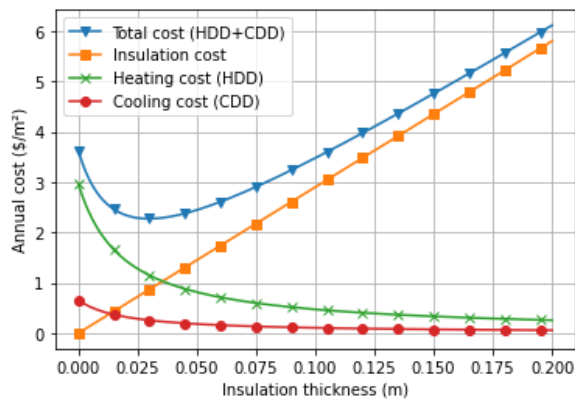
|                                  |       |                   |       |                                   |
|----------------------------------|-------|-------------------|-------|-----------------------------------|
| Thermal conductivity (EPS)       | k     | W/mK              | 0.032 | Expanded polystyrene conductivity |
| Thermal conductivity (rock wool) | k     | W/mK              | 0.040 | Rock wool conductivity            |
| Insulation cost (EPS)            | $C_i$ | \$/m <sup>3</sup> | 29    | Cost of EPS insulation            |

## Results and Discussion

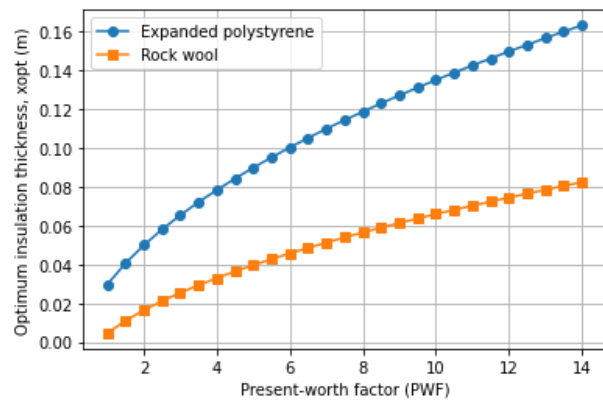
### 1. Effect of insulation thickness on annual energy cost

The variation of annual energy cost with insulation thickness is presented in Fig. 1. The total annual cost consists of three main components: heating cost, cooling cost, and insulation investment.

As insulation thickness increases, the thermal resistance of the wall increases, which reduces heat transfer between indoor and outdoor environments. Consequently, both heating and cooling energy demands decrease significantly. This reduction is reflected in the decreasing heating and cooling cost curves shown in Fig. 1.



**Fig. 1.** Variation of annual heating, cooling, insulation, and total costs with insulation thickness



**Fig. 2.** Variation of optimum insulation thickness with the present worth factor (PWF) for expanded polystyrene and rock wool.

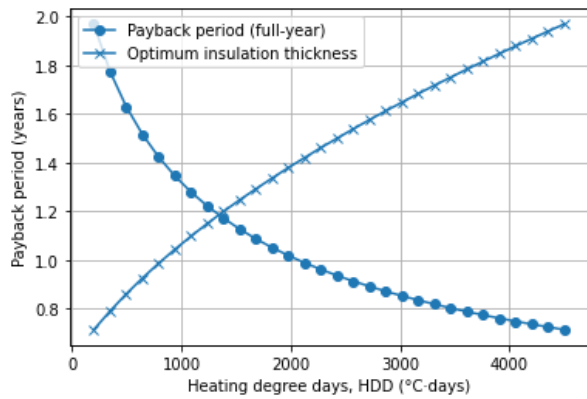
However, insulation cost increases linearly with insulation thickness because it is directly proportional to the amount of insulation material used. As a result, the total annual cost initially decreases due to reduced energy consumption and then increases due to higher insulation investment. The minimum point of the total cost curve corresponds to the optimum insulation thickness, which represents the most economically efficient insulation level.

### 2. Influence of present worth factor on optimum insulation thickness

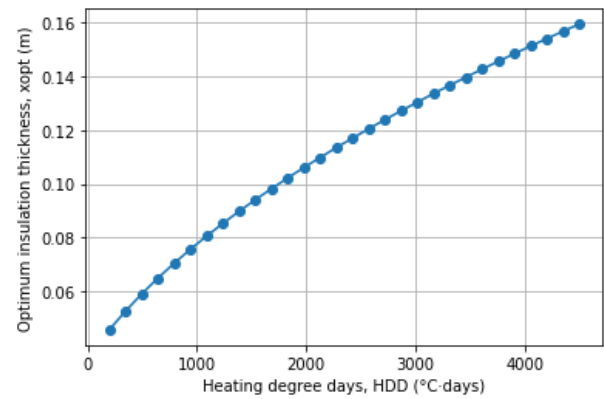
The effect of economic conditions on insulation performance is illustrated in Fig. 2, which shows the variation of optimum insulation thickness with the present worth factor (PWF) for two insulation materials: expanded polystyrene and rock wool. The results indicate that optimum insulation thickness increases with increasing PWF values. A larger PWF corresponds to higher future energy costs or longer system lifetimes, which increases the economic benefits of insulation. Among the two insulation materials investigated, expanded polystyrene exhibits larger optimum insulation thickness values compared to rock wool. This behaviour can be attributed to the lower thermal conductivity and lower material cost of expanded polystyrene, which makes thicker insulation economically favourable.

### 3. Effect of climatic conditions on insulation performance

The influence of climatic conditions is presented in Fig. 3, where the variation of payback period and optimum insulation thickness with heating degree days (HDD) is shown. As HDD values increase, the heating demand becomes larger, which increases the energy savings achieved through insulation. Consequently, the optimum insulation thickness increases with increasing HDD.



**Fig. 3.** Variation of payback period and optimum insulation thickness with heating degree days (HDD).



**Fig. 4.** Relationship between heating degree days (HDD) and optimum insulation thickness.

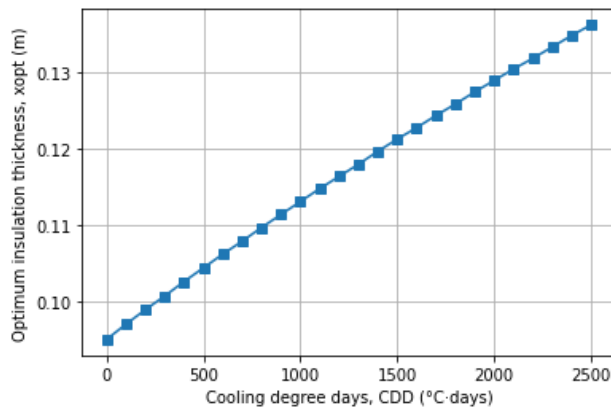
At the same time, the payback period decreases significantly as HDD increases. This occurs because higher heating demand results in larger annual energy savings, allowing the insulation investment to be recovered more quickly. These results demonstrate that insulation becomes more economically attractive in colder climates.

#### 4. Variation of optimum insulation thickness with heating degree days

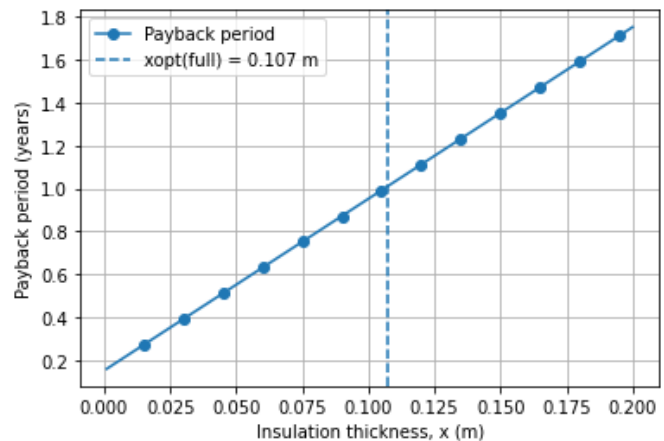
The relationship between heating degree days and optimum insulation thickness is further illustrated in Fig. 4. The results show a clear increasing trend between HDD and optimum insulation thickness. In regions with higher heating demand, thicker insulation layers are required to reduce heat losses effectively. Therefore, optimum insulation thickness increases gradually with increasing HDD values. This behaviour highlights the importance of considering local climatic conditions when designing insulation systems for buildings.

#### 5. Influence of cooling degree days

The impact of cooling requirements on insulation design is presented in Fig. 5, which shows the variation of optimum insulation thickness with cooling degree days (CDD).



**Fig. 5.** Variation of optimum insulation thickness with cooling degree days (CDD).



**Fig. 6.** Variation of payback period with insulation thickness.

The results indicate that optimum insulation thickness increases as CDD increases. This is because insulation reduces not only heating loads but also cooling loads by limiting heat gains through the building envelope. Therefore, insulation plays an important role in improving building energy efficiency in both cold and warm climates.

#### 6. Economic performance and payback analysis

The economic performance of insulation is analysed in Fig. 6, which presents the variation of payback period with insulation thickness. The payback period initially decreases as insulation thickness increases because energy savings increase. However, after a certain point, further increases in insulation thickness lead to diminishing energy savings while insulation investment continues to

increase. Consequently, the payback period begins to increase again for very large insulation thickness values.

The vertical dashed line shown in Fig. 6 represents the optimum insulation thickness obtained from the life-cycle cost analysis. At this point, the insulation system achieves the best balance between investment cost and energy savings.

### Conclusions

This study developed a comprehensive thermo-economic model for determining the optimum insulation thickness of building walls by simultaneously considering heating and cooling energy demands. Unlike conventional approaches based solely on heating degree days (HDD), the proposed model incorporates both HDD and cooling degree days (CDD), providing a more realistic assessment of annual building energy performance. The results showed that the optimum insulation thickness for the investigated climatic conditions (HDD = 2022 °C·days, CDD = 650 °C·days) was approximately 0.07–0.09 m for expanded polystyrene, while slightly lower values were obtained for rock wool due to its thermal and economic characteristics. Applying the optimum insulation thickness reduced annual energy costs by approximately 35–50% compared with an uninsulated wall.

Life-cycle cost analysis indicated that insulation investments can typically be recovered within 3–7 years, confirming the economic viability of thermal insulation. Climatic conditions were found to have a significant influence on insulation design. Higher HDD values increased the optimum insulation thickness and reduced the payback period due to greater heating energy savings. Similarly, increasing CDD values led to larger optimum insulation thicknesses as a result of reduced cooling loads. The developed model provides a practical tool for determining economically optimal insulation thickness under different climatic conditions. Overall, proper insulation design improves thermal comfort, reduces energy consumption and operating costs, and supports sustainable building energy management.

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